Advanced Electric Orbit-Raising Optimization for Operational Purpose

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Abstract: *In recent years electric orbit-raising became an important driver for commercial telecommunication platforms. Clearly, exploiting the high specific impulse of the electric propulsion system is a benefit in propellant consumption when comparing with pure chemical engines. Due to the small thrust magnitudes of the electric propulsion system a transfer from geostationary transfer orbit to geosynchronous equatorial orbit requires several weeks or months of continuous low-thrust acceleration. Furthermore certain issues have to be considered in the trajectory planning like collision avoidance, eclipse handling or other operational aspects as navigation accuracy or ground station contact. Besides, restrictions and limitations may also arise from the spacecraft subsystems. A closed loop guidance, navigation and control algorithm is typically used to simulate these behaviors while the used spacecraft trajectory is simpler since complex mission constraints are not considered.*
This paper presents a new approach where complex mission constraints are already considered in the trajectory optimization, for example slew rate limitations. Such an optimized maneuver plan can be directly used for spacecraft operations simplifying the overall process. Using nonlinear programming to optimize the attitude profile in combination with detailed modelling of complex mission constraints and limitations of the spacecraft model is essential. One example of an electric orbit-raising is shown to demonstrate the capabilities and benefits of the introduced approach.

Keywords: *Electric Orbit-Raising, Re-Optimization of Orbit Transfers, Operational Chain, Slew Rate Optimization, Nonlinear Programming.*

1. Introduction

Telecommunication satellites located in the Geostationary Equatorial Orbit (GEO) are typically not directly placed there by the launch vehicle. The satellites are often injected in a Geostationary Transfer Orbit (GTO) and then transferred to the GEO using their own onboard propulsion system. State of the art for the GTO to GEO transfer is still the chemical propulsion. Just recently few satellites transferred or are transferring to GEO using Electric Propulsion (EP), since it is very attractive to exploit their high specific impulse reducing the propellant mass of the orbit transfer. Since the total spacecraft mass is reduced this yields launch vehicle cost reductions. Further, Electric Orbit-Raising (EOR) is now available for most telecommunication satellite platforms or at least under development.
But electric orbit-raising requires much more complex maneuver sequences than what is needed for pure chemical transfers. Since EP provides only small thrust magnitudes in comparison to chemical propulsion, the transfer lasts many months. A careful planning of the spacecraft attitude maneuvers is required in advance to fulfill this mission. For example, during the transfer any

crossing of the GEO ring poses a certain collision risk with high value assets. Thus, the precomputed transfer trajectory has to avoid crossings of the GEO ring. Further, ground station visibility might be considered for transfer planning as well as limitations and constraints related to different spacecraft subsystems, such as eclipse handling, power generation, storage and consumption, or EP firing limitations in general [10]. Other possible limitations are related to the attitude of the spacecraft or consider environmental aspects like the radiation dose.

Up to now, a closed loop GNC (Guidance, Navigation and Control) algorithm is used to simulate most of the abovementioned aspects. Typically, a reference trajectory does only include basic aspects like perturbations, but not more complex issues like restricted slew rates or limitations in thruster firings. They are only included in the GNC loop to simulate the real spacecraft behavior. A more advanced concept of trajectory optimization of electric orbit-raisings and a closed loop GNC algorithm was presented in [9].

But an optimized orbit transfer as reference trajectory under consideration of the aforementioned model and mission issues and without need of a GNC algorithm would be a benefit for the spacecraft operations since the processing is simplified. Further, it is an important step towards full spacecraft autonomy. This paper shows the capabilities of very sophisticated low-thrust orbit transfers with increased optimality to reduce the need of GNC simulations.

For the planning and computation of the trajectory it is essential to model the real dynamics of the spacecraft. The next chapter will introduce about the spacecraft dynamics in more details. Furthermore typical perturbations which are essential for low-thrust orbit transfers are discussed. In the following chapter the operational concept is focused. This paper details the proposed concept of an operational chain including trajectory optimization as already introduced in [8] and detailed in [10]. The required components are explained and highlight is given on the approach to identify the current location on the reference trajectory. An optimal low-thrust orbit transfer applying the proposed operational concept is presented next. It includes the initial reference trajectory and the results of the re-optimized trajectory as part of the operational chain.

2. Model Dynamics

To compute or optimize the motion and attitude of a spacecraft it is important to describe their representation. Different methods exist to describe translational and rotational spacecraft states and almost every method has certain advantages and disadvantages. This chapter presents dynamic system as well as the perturbations acting on the spacecraft.

2.1. Translational Dynamics

First the translational equations of motion are presented. Newton's law of gravitation states that any two objects of mass m and M attract each other. Assuming the larger mass M is fixed in the inertial space and $m \ll M$ the acceleration vector $\ddot{\mathbf{r}}$ of mass m is

$$\ddot{\mathbf{r}} = -\frac{\mu}{\|\mathbf{r}\|^3}\mathbf{r} + \mathbf{a} \tag{1}$$

where μ is the standard gravitational parameter of mass M, $\mathbf{r}$ is the position vector from M to m and $\mathbf{a}$ is the disturbing acceleration vector. The latter one is required to include disturbing accelerations such as the thrust acceleration or third body gravitational perturbations.

Position vector **r** and velocity vector **v** of the spacecraft are often represented in Cartesian elements. They are transformed into a set of orbital elements. Keplerian orbital elements (semimajor axis a, eccentricity e, inclination i, argument of periapsis ω, right ascension of ascending node Ω and true anomaly ν) suffer from three singularities. First, the line of apsis is undefined for circular orbits where the eccentricity is zero. Second, with an inclination of 0° or 180° the line of nodes is undefined since the orbital plane lies in the x-y plane of the inertial frame. And third, the semimajor axis is not continuous for parabolas where the eccentricity equals one. This is a crucial aspect for the dynamics in case elliptic and hyperbolic orbits are involved in the trajectory.

As suggested in [4], a set of modified equinoctial orbit elements is introduced to remove the shortcomings:

- Equinoctial element p is the semi-latus rectum
- Equinoctial element f and g represent the eccentricity vector
- Equinoctial element h and k represent the inclination vector
- Equinoctial element L is the true longitude of the spacecraft position

This set of orbital elements is very suitable for trajectory optimization because the results are more precisely and the convergence is better toward the Keplerian elements. Also the required time for the optimization is less. The modified equinoctial elements are defined by

$$p = a(1 - e^2) \tag{2}$$
$$f = e\cos(\omega + \Omega) \tag{3}$$
$$g = e\sin(\omega + \Omega) \tag{4}$$
$$h = \tan\left(\frac{i}{2}\right)\cos\Omega \tag{5}$$
$$k = \tan\left(\frac{i}{2}\right)\sin\Omega \tag{6}$$
$$L = \Omega + \omega + \nu \tag{7}$$

and the Keplerian elements are defined by the inverse transformation

$$a = \frac{p}{(1 - f^2 - g^2)} \tag{8}$$
$$e = \sqrt{f^2 + g^2} \tag{9}$$
$$i = 2\tan^{-1}\sqrt{h^2 + k^2} \tag{10}$$
$$\omega = \tan^{-1}\frac{g}{f} - \tan^{-1}\frac{k}{h} \tag{11}$$
$$\Omega = \tan^{-1}\frac{k}{h} \tag{12}$$
$$\nu = L - \tan^{-1}\frac{g}{f} \tag{13}$$

Next, the disturbing acceleration vector in a rotating frame is introduced:

$$\Delta = \mathbf{R}^{\mathrm{T}}\mathbf{a} \tag{14}$$

where the corresponding transformation matrix **R** is given by

$$\mathbf{R} = [i_r \quad i_t \quad i_n] \tag{15}$$

with

$$i_r = \frac{\mathbf{r}}{\|\mathbf{r}\|} \tag{16}$$

$$i_t = \frac{(\mathbf{r}\times\mathbf{v})\times\mathbf{r}}{\|(\mathbf{r}\times\mathbf{v})\times\mathbf{r}\|} \tag{17}$$

$$i_n = \frac{\mathbf{r}\times\mathbf{v}}{\|\mathbf{r}\times\mathbf{v}\|} \tag{18}$$

defining the rotating frame with respect to the inertial frame. Its origin is located in the center of mass of the spacecraft. Index r indicates the radial component pointing in the same direction as the position vector, whereas index t is the transverse (along-track) component lying in the orbital plane pointing in the direction of flight but not necessarily parallel to the velocity vector. Both axes span the orbital plane. Index n is the normal (cross-track) component being perpendicular to the orbital plane and pointing in the direction of the angular momentum. This rotating coordinate frame is also called RTN or LVLH (local vertical, local horizontal).
Considering the acceleration vector components defined in the rotating frame, the equinoctial dynamics are defined by [8]

$$\dot{p} = \sqrt{\frac{p}{\mu}}\Delta_t \frac{2p}{w} \tag{19}$$

$$\dot{f} = \sqrt{\frac{p}{\mu}}\left\{\Delta_r \sin L + \Delta_t \frac{1}{w}[(w+1)\cos L + f] - \Delta_n \frac{g}{w}[h \sin L - k \cos L]\right\} \tag{20}$$

$$\dot{g} = \sqrt{\frac{p}{\mu}}\left\{-\Delta_r \cos L + \Delta_t \frac{1}{w}[(w+1)\sin L + g] + \Delta_n \frac{f}{w}[h \sin L - k \cos L]\right\} \tag{21}$$

$$\dot{h} = \sqrt{\frac{p}{\mu}}\Delta_n \frac{s^2}{2w}\cos L \tag{22}$$

$$\dot{k} = \sqrt{\frac{p}{\mu}}\Delta_n \frac{s^2}{2w}\sin L \tag{23}$$

$$\dot{L} = \sqrt{p\mu}\left(\frac{w}{p}\right)^2 + \sqrt{\frac{p}{\mu}}\Delta_n \frac{1}{w}(h \sin L - k \cos L) \tag{24}$$

where

$$w = 1 + f\cos L + g \sin L \tag{25}$$

$$s^2 = 1 + h^2 + k^2. \tag{26}$$

2.2. Rotational Dynamics

Also the dynamics of the rotational motion have to be introduced. They are based on the well-known Euler's equations and describe the angular accelerations. The general form of Euler's rotation equations is defined as

$$\boldsymbol{\tau} = \mathbf{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\mathbf{I}\boldsymbol{\omega}) \tag{27}$$

Where $\boldsymbol{\tau}$ is the applied torque vector (also known as moment), $\mathbf{I}$ is the inertia matrix and $\boldsymbol{\omega}$ is the angular velocity vector about the principal axes. The angular velocity vector is the rotation vector of the spacecraft body axes with respect to the inertial frame and given as

$$\boldsymbol{\omega} = \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \tag{28}$$

In literature the components along the three principal axes x, y and z are also known as p, q and r, respectively. Next, the inertia matrix of the spacecraft is defined as

$$\mathbf{I} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix} \tag{29}$$

Assuming two planes of symmetry for the spacecraft, like it is the case for rotational symmetric bodies, yields

$$I_{xy} = I_{xz} = I_{yz} = 0 \tag{30}$$

and Euler's equations are simplified to

$$\tau_x = I_{xx}\dot{\omega}_x + \left(I_{zz} - I_{yy}\right)\omega_y\omega_z \tag{31}$$
$$\tau_y = I_{yy}\dot{\omega}_y + \left(I_{xx} - I_{zz}\right)\omega_x\omega_z \tag{32}$$
$$\tau_z = I_{zz}\dot{\omega}_z + \left(I_{yy} - I_{xx}\right)\omega_x\omega_y \tag{33}$$

In a next step the relationship between the rotation vector and the attitude rates of the spacecraft is shown. First, the spacecraft attitude is defined in the inertial frame with Euler angles and rates according to

- Yaw angle ψ_i and its angular rate $\dot{\psi}_i$
- Pitch angle θ_i and its angular rate $\dot{\theta}_i$
- Roll angle φ_i and its angular rate $\dot{\varphi}_i$.

Next, the body rates about the spacecraft axes are defined as [6]

$$\omega_x = \dot{\varphi}_i - \dot{\psi}_i \sin\theta_i \tag{34}$$
$$\omega_y = \dot{\theta}_i \cos\varphi_i + \dot{\psi}_i \cos\theta_i \sin\varphi_i \tag{35}$$
$$\omega_z = -\dot{\theta}_i \sin\varphi_i + \dot{\psi}_i \cos\theta_i \cos\varphi_i \tag{36}$$

and the inverse transformation to retrieve the Euler angle rates is given by

$$\dot{\psi}_i = \frac{\sin\varphi_i}{\cos\theta_i}\omega_y + \frac{\cos\varphi_i}{\cos\theta_i}\omega_z \tag{37}$$
$$\dot{\theta}_i = \cos\varphi_i\, \omega_y - \sin\varphi_i\, \omega_z \tag{38}$$
$$\dot{\varphi}_i = \omega_x + \sin\varphi_i \tan\theta_i\, \omega_y + \cos\varphi_i \tan\theta_i\, \omega_z \tag{39}$$

Either the body rates or the Euler angular rates can be used as control vector for the optimal control problem. In case of the first one the control needs to be transformed into the Euler angular rates to compute derivatives of the spacecraft attitude. Instead when using the Euler angular rates as control they can be directly used in the equations of motion of the spacecraft attitude.

2.3. Perturbations

As defined in Eq. (1) an additional acceleration is part of the spacecraft dynamics. It includes all perturbing forces acting on the spacecraft during its orbit. For electric orbit-raisings it is mainly the thrust acceleration of the on-board propulsion system. In contrary to natural forces it is actively controlled and long thrust arcs are one of the characteristics of low-thrust orbit transfers. Besides the thrust acceleration, other perturbations have to be taken into account as well such as accelerations due to gravitational fields and non-conservative perturbations.
Gravitational accelerations comprise third body perturbations and the inhomogeneous gravity field of the primary body. Especially the oblateness (J2) of the Earth has to be taken into account since it is one the major perturbations. For orbit transfers around Earth, additional celestial bodies like Moon and Sun might have a strong impact on the satellite dynamics, while other planets can be neglected. However, it is suggested to use the approach initially presented by Battin [2].
The most prominent non-conservative disturbing acceleration is the atmospheric drag. Certainly, its impact on the trajectory is strong on low altitudes. Once the spacecraft travels e.g. above 1000 km of altitude the atmospheric drag becomes very small and is dominated by other perturbations. Atmospheric drag is caused by the particles of the atmosphere and depends on its density as well as the velocity of the spacecraft with respect to the atmosphere.
Since the spacecraft applies solar-electric propulsion it is equipped with large solar panels to collect the solar energy required to operate the electric propulsion system. Thus the mass to area ratio becomes quite small. In such situations the solar radiation pressure (SRP) affects the spacecraft trajectory and has to be considered. SRP is the pressure exerted by the solar radiation on objects within its reach, like satellites and spacecraft in general. Its effect is strongest for objects with small masses and large reference areas.

3. Operational Chain

After the successful launch of the spacecraft the operational phase of the orbit transfer starts. Its goal is to safely bring the satellite from its initial transfer orbit to the desired target location in geostationary ring. Because of the low-thrust character and the long duration of the transfer, a periodic operational process is proposed. It can be on daily, weekly, bi-weekly or monthly basis, or anything between. This cyclic concept was already applied for earlier investigations on re-optimization of perturbed GTO-GEO transfers [8]. One example of a cycle is illustrated in Fig. 1.

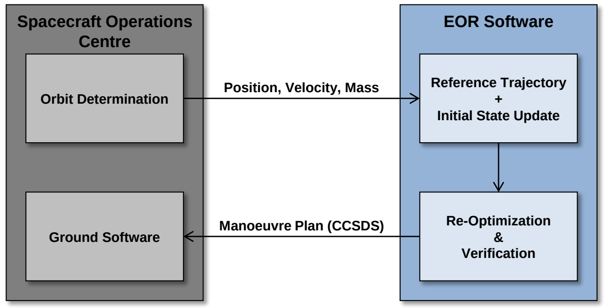

Figure 1. One example of an operational chain for a ground-based navigation concept. The spacecraft operations center (left) is responsible for orbit determination and upload of the optimized maneuver plan to the spacecraft. The optimization software for the EOR scenarios handles the reference trajectory and its re-optimization considering the updated spacecraft states.

An operational chain may involve the following components:

- Orbit determination
- Reference trajectory
- Update of the spacecraft state
- Re-optimization of the trajectory to retrieve the maneuver plan
- Further verification and analysis tasks
- Processing of maneuver plan and upload to satellite
- Wait one period while the spacecraft travels

In every cycle the components of the chain are accomplished. A low-thrust orbit transfer consists of several cycles between initial and target orbit. In other words, the whole trajectory is segmented into smaller parts where each single part covers one period. Since the processing of the components, except the last one, shall be in short time it requires a very good and efficient interaction of the involved hardware, software and personal.

3.1. Orbit Determination

First of all the orbit of the satellite has to be determined. It comprises the position and velocity as well as the mass of the spacecraft. These values are taken as input parameters for the following trajectory optimization (EOR software). State of the art today is ground-based determination of the abovementioned parameters. For future concepts there are already investigations whether this task can be handled by the spacecraft itself. Currently it is one limitation towards full autonomy of spacecraft orbit transfers.

3.2. Reference Trajectory

Further, the software considers a reference trajectory. The one of the first operational cycle is pre-computed before the actual mission. In all other following cycles it is taken from the previous cycle. So the re-optimized trajectory becomes the new reference solution for the following operational cycle. But because of small uncertainties in the initial orbit (e.g. injection

errors) or during the transfer, it is required to update the state of the reference trajectory with the real one of the spacecraft.
For example, in the first cycle the spacecraft is still in its transfer orbit where it was released by the launch vehicle. During the first orbital revolutions all spacecraft systems are activated and checked. Once everything works well the orbit transfer is initiated. Injection errors of the launcher as well as atmospheric drag and other perturbations alter the planned initial orbit which is considered in the reference trajectory. Therefore, an update of the trajectory must be computed taking into account the new and actual spacecraft state.

3.3. Update of Spacecraft State

The next step of the cycle is probably of the most crucial one. Once the current spacecraft state is known, it has to be correlated with the previously computed reference trajectory. This procedure shall identify the point on the reference trajectory where the spacecraft is currently located. Obviously, the actual flown trajectory is different from the pre-optimized reference trajectory. In a perfect world the spacecraft follows exactly the reference trajectory without the smallest deviation. But in reality this is not the case and any deviation from the reference trajectory must be considered.
To identify the current location on the reference trajectory several possibilities exist, such as

- Position
- Velocity
- One, few or all orbital elements
- Flight time
- Julian date
- Spacecraft mass
- Any combination of above-mentioned

Position and velocity can be transformed into a set of orbital elements. It is important to consider the current orbit properties since it defines the remaining transfer duration. Further, flight time and/or Julian date strongly impact the occurrence of eclipses, since they change with the seasons [10]. And the mass of the spacecraft impacts the thrust-to-mass ratio and therefore the spacecraft acceleration.
Once the current location on the reference trajectory is identified, the part of the trajectory already travelled by the spacecraft is removed and the remaining part becomes the new reference. Next, the initial state is updated with the real spacecraft state and then the trajectory is re-optimized. This approach is much faster than to optimize from “scratch” the remaining transfer, since the already known attitude history requires only small variations to be compliant with the constraints.

3.4. Re-Optimization

When talking about optimization of a low-thrust orbit transfer it is meant to solve the optimal control problem. Very efficient is a direct transcription of the optimal control problem into a nonlinear programming (NLP) problem by discretization. Several discretization schemes are available, such as trapezoidal or the higher order Hermite-Simpson discretization methods [3, 5]. The trapezoidal discretization method is faster in computation but less robust than Hermite-

Simpson. This approach is known as direct transcription by collocation. In this case the nonlinear optimization problem becomes quite huge with tens or hundreds of thousands of parameters and constraints. Nevertheless, a NLP problem is easier to solve than a boundary-value problem since it is sparse. Here, the NLP is solved by using sequential quadratic programming.

Especially under consideration of tight accuracy and fidelity requirements for achieving optimality in sense of propellant consumption and transfer duration, an efficient and fast optimization algorithm is required, such as the proposed direct collocation.

When re-optimizing the remaining transfer, the proper part of the reference trajectory is taken and its initial state updated with state identified by orbit determination. Further, the reference trajectory is used as initial guess.

To constrain for example slew rates or other aspects of the spacecraft subsystems, the optimization problem is transformed into two phases. In the first phase, which covers the period of the next cycle (for example one week), a very sophisticated model is used including the rotational dynamics of the spacecraft as described in section 2.2. Besides, other aspects like battery capacity can be included in the dynamics. Since rotational dynamics are changing very fast, this optimal control problem needs a denser discretization grid. This results in much more optimizable parameters and constraints of the NLP. However, the second phase covers the remaining part to the final orbit. It does only include the translational dynamics but not the rotational ones. In general, the second phase has the same capabilities of what was presented in [10]. For example, perturbations, eclipse handling (see Fig. 2) and geometrical constraints like GEO ring crossing avoidance are included. While phase two uses an advanced model to describe the spacecraft dynamics and constraints, phase one improves it even more taking into account also the rotational dynamics.

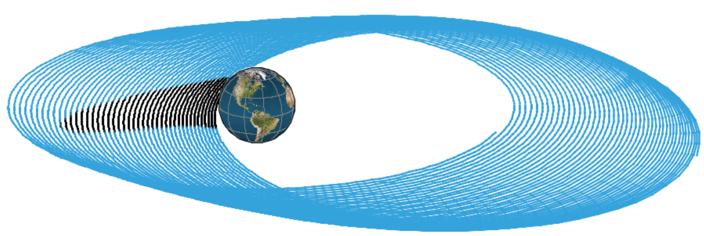

Figure 2. Eclipses (black) during a multi-revolution low-thrust transfer (blue) illustrated in inertial frame

In principle, in the next cycle the phase one is travelled by the spacecraft and this part will be then removed from the reference trajectory. What remains is updated with the new spacecraft state, transformed into a two-phase problem and re-optimized.

3.5. Verification and Analysis

Once the trajectory is optimized the computed maneuver plan has to be verified to meet all constraints and requested conditions, because there are further aspects for electric-orbit raisings that might be crucial [10]. First of all, during its transfer the spacecraft has to avoid any possible collision. A collision risk exists for other objects like active/inactive satellites or upper stages and thousands of space debris. All those objects pose a serious threat for spacecraft with large solar arrays like it is the case for electric orbit-raising vehicles. A post-processing analysis is

enough when no threats are identified. Otherwise it has to be taken into account in the re-optimization of the trajectory, for example as geometrical constraint.
Another aspect is the ground station visibility. At least at the end of the current operational cycle it is required to have contact between the spacecraft and the ground station. It is necessary for two reasons at the minimum: first, to determine the current orbit and mass of the spacecraft. And second, the new maneuver plan needs to be uploaded to the spacecraft.

3.6. Maneuver Plan

In the final step of the operational chain the maneuver plan for the next period (one week) is extracted from the optimal trajectory, processed into the ground software and uploaded to the satellite. After one period the next cycle starts again with the orbit determination. But this part of the trajectory already travelled is removed from the reference trajectory.
In principle, the re-optimization process either optimizes the next intermediate cycle or the remaining transfer to the target (last cycle). Obviously, in the first case some margins for the propulsion system are required. In [8] it was shown that a completely unperturbed GTO-GEO transfer, without any perturbations like third bodies or solar radiation pressure, can be used as reference trajectory for a simulated operational chain process. Taking into account a margin for the propulsion system, the spacecraft could follow the reference trajectory and compensate all disturbances caused by J2, SRP and third bodies, which have been considered in the dynamics of the re-optimization.

4. Example

This chapter presents one example of a low-thrust multi-revolution orbit transfer applying the newly introduced operational concept. A trajectory is considered from a geostationary transfer orbit with an inclination 27 degrees. The target for the e.g. telecommunication satellite is the geosynchronous equatorial orbit. In Tab. 1 the orbital parameters are summarized for both initial and final orbit. Since the mission start date is around 21st of March, the right ascension of ascending node of the initial orbit is chosen 0 degree. Thus the apoapsis is located in direction of Sun while the periapsis is located in the shadow of Earth. Further, the spacecraft starts its transfer while located in apoapsis.
A mass of 1,000 kg and a thrust magnitude of 150 mN were assumed to represent a typical thrust-to-mass ratio of GTO-GEO transfer satellites. It results in about 6 month of transfer duration and considering a specific impulse of 2,000 seconds the propellant consumption is about 12.5% of the initial mass.

Table 1. Orbital parameters of initial orbit (GTO) and target orbit (GEO)

	Initial Orbit	Target Orbit
Periapsis Altitude	250 km	35,786 km
Apoapsis Altitude	35,786 km	35,786 km
Inclination	27 deg	0 deg
Arg. Of Periapsis	178 deg	undefined
Right ascension	0 deg	undefined
True anomaly	180 deg	undefined

For the trajectory optimization the software LOTOS is used [1]. It computes fully automatic initial guesses based on analytic steering laws and optimizes the transfer trajectory subject to several constraints and objectives. This software tool does also automatically process the reference trajectory and identifies the current spacecraft location on the reference trajectory. Everything is setup through the built-in batch processing tool Batch-Mode Inspector.

The orbit determination is simulated by extracting the orbit after one cycle from the reference trajectory. Then, a deviation is applied to the orbital elements to pretend a small offset from the nominal, which might be the case in reality because of for example thruster over- or underperformance.

The given example is for the second cycle. It considers that the spacecraft already travelled 7 days to GEO, the duration of cycle #1. For the orbit determination, a spacecraft state (position, velocity, mass) was extracted from the reference trajectory after 7 days of transfer duration and perturbed to simulate a real spacecraft deviation from the nominal trajectory. Using the perturbed initial state of cycle #2 as actual spacecraft state, the software LOTOS identifies the location on the reference trajectory which is closest to the actual one (values are presented in Tab. 2).

Table 2. Actual spacecraft state (estimated) and identified orbit on reference trajectory for operational cycle #2

	Estimated Actual Spacecraft State	Identified Orbit on Reference Trajectory
Periapsis Altitude	636 km	621 km
Apoapsis Altitude	36,930 km	36,865 km
Inclination	25.475 deg	25.527 deg
Spacecraft Mass	995.370 kg	995.557 kg

As described in the previous chapter, the optimal control problem is split into two phases. While the first segment covers the duration of the second operational cycle i.e. 7days, the second segment covers the remaining transfer to the target orbit. Note phase 1 does include the rotational dynamics of the spacecraft to be able to constrain the spacecraft body rates. It increases the number of parameters and constraints as well as the complexity of the optimal problem. However, an optimal solution is found within few minutes on a standard desktop computer.

Figure 3 shows the evolution of the orbital elements semimajor axis, eccentricity and inclination for the whole transfer. The already travelled trajectory of the first 7 days is indicated as dotted line. Cycle #2 is the small part between "[" and "][", and the remaining part comprises several upcoming cycles. In Fig. 4 the evolution of the thrust vector components in the rotating RTN frame is shown.

The remaining transfer of about 182 days, the first 7 days are cycle #2, will be handled in similar way until the spacecraft finally reaches its target orbit. In total, about 27 cycles are required for this GTO to GEO transfer. This number can be reduced in case the duration of a cycle is extended to e.g. 14 days.

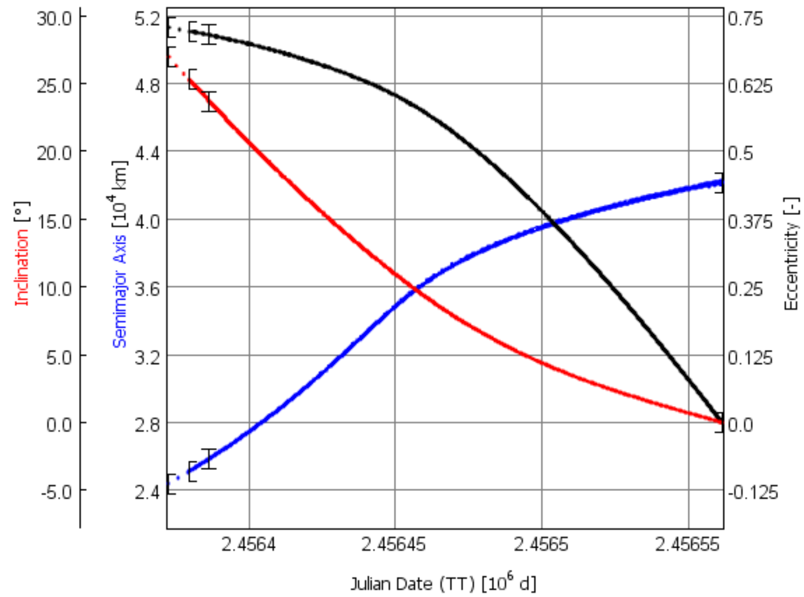

Figure 3. Evolution of orbital elements semimajor axis (blue), eccentricity (black) and inclination (red) after re-optimization of the trajectory. The dotted lines at the beginning of the transfer indicate the already travelled part of trajectory (cycle #1). Directly next to it the second cycle can be identified within the symbols "[" and "][".

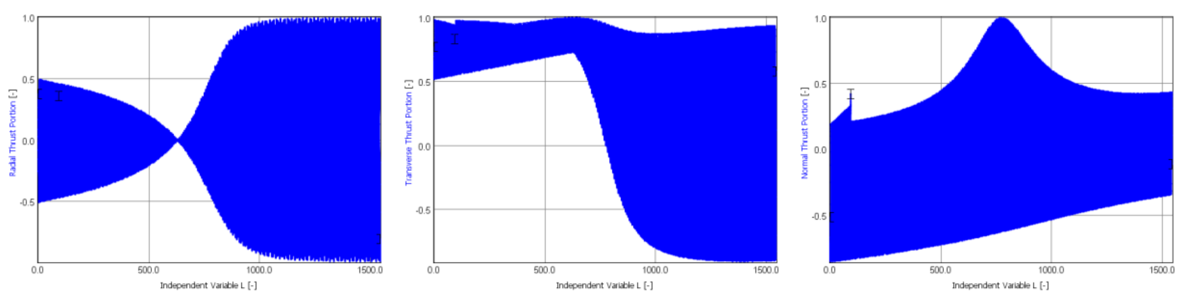

Figure 4. Evolution of the radial (left), transverse (middle) and normal (right) thrust vector components in the rotating frame. In the left part of each figure the cycle #2 is the area between "[" and "][".

5. Conclusions

This paper introduced a new approach for the operation of low-thrust orbit transfers. It is possible to consider complex constraints such as slew rate limitations in the optimization process. Further, it was shown that 6 month lasting low-thrust multi-revolutions transfers can be constrained in spacecraft body rates. While keeping the previously computed reference trajectory, it is possible to re-optimize the remaining transfer within few minutes, even when the actual spacecraft state deviates from the nominal. The overall processing for operations is simplified since complex GNC simulations are not mandatory.

The presented concept involves a sophisticated algorithm to identify the location on the reference trajectory according to the current spacecraft state. Furthermore the shown concept is one

essential step towards supervised autonomy of orbit transfer spacecraft because most aspects and required working steps like identification of the spacecraft location and optimization are already automated. Once it has shown its reliability the next incremental step can be already towards full autonomy. Obviously, it requires progress in on-board orbit determination through global navigation systems.
A comparison with a closed loop GNC simulation would be very interesting by means of performance (e.g. CPU time) and transfer characteristics (transfer duration, propellant consumption). Further it could identify the benefits and drawbacks of both methods.

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